

Engineering atom-atom thermal entanglement via two-photon process

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Abstract

We study the system that two atoms simultaneously interact with a single-mode thermal field via different couplings and different spontaneous emission rates when two-photon process is involved. It is found that we indeed can employ the different couplings to produce the atom-atom thermal entanglement in two-photon process. The different atomic spontaneous emission rates are also utilizable in generating thermal entanglement. We also investigate the effect of the cavity leakage. To the initial atomic state $|ee\rangle$, a slight leakage can relieve the restriction of interaction time and we can obtain a large and steady entanglement.

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1 Introduction

Entanglement plays an important role in respect that it is a valuable resource in quantum information processing such as quantum teleportation[?], quantum computation[2] and quantum cryptography[3], *etc.* Several schemes have been proposed to prepare purified and distilled entangled state both theoretically and experimentally[4]. Although the interaction between a quantum system and its surroundings can result in inevitable decoherence of the quantum system, people have recognized that we can employ the interaction to generate entanglement[5].

The two-atom entangled states are widely studied in cavity QED[6, 7, 8, 9, 10]. In cavity QED, the dissipation in the model of atoms interacting with magnetic field generally includes two aspects: the cavity leakage through which the intra-cavity magnetic field can exchange information with its environmental

noise, the atomic spontaneous emission that is induced by vacuum fluctuation effect. In the sense of using the impact of environmental noise, the noise-assisted entanglement schemes have been put forward by many authors[9, 10, 11, 12, 13, 14, 15]. Plenio and co-work have developed schemes that involves continuous monitoring of photons leaking out of the cavity to entangle atoms one of which is initially excited[9]. In Ref. [10], the author studied the interaction of a thermal field with a two-qubit system that initially prepared in separable states. They demonstrated that entanglement of atom-atom can arise depending on initial preparation of the atoms. Also in Ref. [11], the entanglement of atom-atom can be generated through interaction of atoms with cavity mode coupled to a white noise. Their entanglement can be maximized for intermediate value of noise intensity and initial value of spontaneous rate. In these studies, the couplings of atoms to field are confined to be equal. In fact, the coupling rate g between atomic internal levels and the cavity mode depends on the atom's position $\mathbf{r}(x, y, z)$ [16]. The atoms can not be localized precisely even by employing cooling technology and trapping potential schemes. So, it is practically necessary to address the question: how will the entanglement be when two atoms differently couple to a single model field? In Ref.[14], our group had shown that different couplings can really assist the induce of entanglement in one-photon process.

On the other hand, the atomic spontaneous emission rate is also related to atoms's position[16]. In real experimental scenario, the atoms's position $\mathbf{r}(x, y, z)$ not only dominates the atom's coupling strength to the field, but also determines the amount of atomic spontaneous emission rate. It has already been reported that the resonant cavity which was made of two spherical niobium mirrors can enhance or suppress single atomic spontaneous emission by adjusting atom position Z (the distance from median plane of cavity)[17, 18]. But theoretically, atomic spontaneous emissions have been assumed to be equal or even been ignored, and the spontaneous emission has been disliked because

of its impact on the entanglement[12]. Up to now we have not found the study that two atoms spontaneous emission rates are not the same. Addition to that, the two-photon process is a kind of important one which may show different properties from the case of one photon in quantum information processing, for example, it has been found that the atom-atom entanglement induced by thermal field in two-photon process is larger than that in one-photon process[19].

In this paper, considering the two-photon process, we aim to study the two atoms simultaneously interacting with a single-mode cavity field with different couplings and different spontaneous emission rates. We find that in two-photon process we indeed can employ the different couplings to produce the atom-atom thermal entanglement. If the atoms spontaneously emit inevitably, the different spontaneous emission rates is utilizable in generating thermal entanglement. We also investigate the effect of the cavity leakage. To the initial atomic state $|gg\rangle$, the cavity dissipation should be suppressed as possible as we can, but to the initial atomic state $|ee\rangle$, we can keep a slight leakage to relieve the restriction of interaction time so that we can obtain a large and steady entanglement.

2 The interaction of two-atom system and the Master equation

The two two-level identical atoms (atom a and atom b) are supposed to interact with a single mode cavity field which is in a thermal equilibrium with its environment characterized in terms of an mean photon number $N = (e^{-\frac{\hbar\omega}{\kappa_B T}} - 1)^{-1}$, and T is the environmental temperature. We assume the excited atom can transit from its upper state to its lower state and emit two photons. So that, the atomic transition frequency ω_0 doubles the field frequency ω . The *Hamiltonian*

under the rotating wave approximation is

$$H = \omega_0 \sigma_a^z + \omega_0 \sigma_b^z + \omega a^\dagger a + \sum_{i=a,b} g_i (a^2 \sigma_i^+ + a^{\dagger 2} \sigma_i^-). \quad (1)$$

Where a and a^\dagger represents annihilation and creation operator of cavity mode respectively. The operators σ_i^- and σ_i^+ denote atomic transition operators of atom i . The coupling constant for two-photon transition between atom i and the cavity mode is g_i . In the interaction picture, the *Hamiltonian* is

$$H_I = g_a a^2 \sigma_a^+ + g_b a^{\dagger 2} \sigma_b^-. \quad (2)$$

For the sake of the two couplings' diversity, the following transformation is preferred

$$g = \frac{g_a + g_b}{2}, \quad r = \frac{g_a - g_b}{g_a + g_b}, \quad (3)$$

where the r is in the range of 0 and 1.

For generality, we assume the intra-cavity system can exchange information with thermal environment due to cavity dissipation and atomic spontaneous emission. The time evolution of the global system (atoms+cavity mode) is governed by the master equation

$$\dot{\rho} = -i[H, \rho] + L(\rho). \quad (4)$$

The *Liouvillean* that describes the atomic spontaneous emission and the interaction of the cavity mode with the thermal environment in a leaky cavity is written as[20]

$$\begin{aligned}
L(\rho) = & - \sum_{i=a,b} [(\bar{n}+1)\Gamma_i(\sigma_i^+\sigma_i^-\rho + \rho\sigma_i^+\sigma_i^- - 2\sigma_i^-\rho\sigma_i^+) \\
& n\Gamma_i(\sigma_i^-\sigma_i^+\rho + \rho\sigma_i^-\sigma_i^+ - 2\sigma_i^+\rho\sigma_i^-)] \\
& -\kappa(\bar{n}+1)(a^+a\rho + \rho a^+a - 2a\rho a^+) \\
& -\kappa\bar{n}(aa^+\rho + \rho aa^+ - 2a^+\rho a),
\end{aligned} \tag{5}$$

where the terms including κ in $L(\rho)$ are interpreted as the coupling strength of cavity mode to the external thermal field, Γ_i is the spontaneous emission rate of atomic i ($i = a, b$). Since Γ_a can be different from Γ_b , we adopt the transformation similar to Eq. 3

$$\Gamma = \frac{\Gamma_a + \Gamma_b}{2}, \gamma = \frac{\Gamma_a - \Gamma_b}{\Gamma_a + \Gamma_b}. \tag{6}$$

The Wootters concurrence that has been proved effective in presenting the entanglement degree of two qubits is written as[21]

$$C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \tag{7}$$

where the λ_i are non-negative real square roots of the eigenvalues of the *Hermitian* matrix $\sqrt{\tilde{\rho}}\tilde{\rho}\sqrt{\tilde{\rho}}$ in decreasing order with $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. No matter what ρ stands for a pure or a mixed entangled state, Wootters concurrence is available. The amount of entanglement measured by concurrence on the basis of different initial atomic states will be numerical calculated in next two sections.

3 Atom-atom thermal entanglement under different couplings and different spontaneous emission rates

We assume that the single mode cavity field is initially in a thermal field state. Due to the cavity leakage when the cavity is in a thermal equilibrium with its

environment, the cavity field is in a mixture of Fock states. So, the cavity field initially takes the form

$$\rho_f(0) = \sum_n |n\rangle \langle n| \frac{N^n}{(1+N)^{n+1}}. \quad (8)$$

Firstly, we study the effect of relative coupling difference r on the two-atom entanglement when two-photon process is involved. The chosen parameters are $g = 1$, $N = 1.5$, $\Gamma_i = 0$ and $\kappa = 0$. We have cut off the intra-cavity photon number at a value of 5 which is precise enough in respect that $\frac{N^n}{(1+N)^{n+1}}$ is a decreasing function of photon number. Fig.1a shows the entanglement as a function of relative coupling difference r and time t in the case of the two atoms are initially in $|ee\rangle$, and Fig.1b is the same as Fig. 1a except that the two-atom are initially in $|gg\rangle$. In Ref.[14], the authors could not find the entanglement induced by thermal field in two-photon process when the initial atomic state is $|ee\rangle$ if two couplings are equal. Fig. 1a also shows there is no entanglement when $r = 0$. But if $r \neq 0$, in some region one can find entanglement. So, in two photon process the different couplings can also benefit to produce entanglement. Comparing Fig.1a with Fig.2 of Ref.[14], we find that the entanglement in one-photon process (Fig.2 of Ref.[14]) exists in some discontinuous small areas in terms of r and t , i.e., for different r entanglement may appear in different interval of time, however, Fig. 1a shows that the entanglement appears in some continuous regions, that is to say, in the relative slowly varying region of time the entanglement keeps its value even the relative large change of r . This property will be more obvious when the two-atom are initially in $|gg\rangle$, which is shown in Fig. 1b. The behavior that the entanglement varies with r and t is very interesting, and the entanglement can exist in almost the same interval of time for different r . For example, in the region $0.8 < t < 1.4$, the entanglement increases to a maximum slightly with the increasing of r from 0 to 0.8, then it decreases to zero. In other words, if we control the interaction time in the interval $0.8 < t < 1.4$, we need not have to care much about whether

the two atoms are in the same position or not. So, in two-photon process it is experimentally not necessary to control the position precisely, especially when the initial atomic state is $|gg\rangle$.

Then, we consider the effect of different spontaneous emission rates of two-atom on the amount of entanglement. We show the typical result of atom-atom entanglement as a function of the difference between two emissions and the noise intensity (mean photon number of thermal environment) in Fig. 2a and Fig. 2b corresponding to atomic initial states $|ee\rangle$ and $|gg\rangle$ respectively. The chosen parameters are $\kappa = 0$, $g = 1$, $r = 0.3$, $\Gamma = 0.2$ and $t = 1$ in Fig. 2a and $\kappa = 0$, $g = 1$, $r = 0.3$, $\Gamma = 0.02$ and $t = 1$ in Fig. 2b. From Fig. 2, we see that the amount of entanglement when difference of two emissions equals to zero, i.e. $\Gamma_a = \Gamma_b$, is not the best case of atom-atom entanglement. The maximum value of entanglement is monotonously increased by increasing the relative difference of two spontaneous emissions γ . For example, the entanglement when $\gamma = 1$ in Fig. 2a is about 1.5 times of that when $\gamma = 0$, and in Fig. 2b, the entanglement when $\gamma = 1$ is even enhanced to be about 6 times of that when $\gamma = 0$. And Fig.2a shows that the entanglement decreases monotonously with the increasing of mean photon number which is also observed in Ref.[14]. One can also observe that the entanglement can be increased by increasing mean photon number in some region in Fig. 2b. This is because that the two atoms initially in $|gg\rangle$ can not be entangled when they interact with vacuum field state. With the increasing of mean photon number in some extent, the entanglement is gradually increased to a maximum. One can observe that the amount of entanglement with spontaneous emission is quite different from that in Fig. 1(without spontaneous emission). When there is atomic spontaneous emission, even this emission is very weak, the amount of entanglement will be much weakened. As mentioned above, in any experimental scenario, the atomic spontaneous emissions can hardly be all kept as zero. Therefore, any entanglement that has been realized experimentally is in fact smaller than theoretical result of ideal model. To investigate the

influence of atomic spontaneous emission on the atom-atom entanglement, the authors in Ref. [12] assume two atoms have a same spontaneous emission Γ in a vacuum cavity. Their results show that the amount of entanglement is a monotone decreasing function of Γ . While, if there is inevitable spontaneous emission in experiment, the difference of spontaneous emission rates can also assist atom-atom entanglement.

4 The effects of dissipation on the atom-atom thermal entanglement

We now turn to consider the situation when cavity keeps on leaking throughout the whole evolution. Fig.3 shows the atom-atom entanglement changing with cavity dissipation κ and time t . Fig. 3a and Fig. 3b are corresponding to the initially atomic state $|gg\rangle$ and $|eg\rangle$ respectively. The chosen parameters in both cases are $N = 1.5$, $\Gamma_a = \Gamma_b = 0$ and $r = 0.1$. When the two-atom are initially in $|gg\rangle$, the amount of entanglement is a monotone decreasing function of cavity decay. With the cavity dissipation increasing, the entanglement decreases rapidly. It denotes that we should depress the cavity dissipation as possible as we can if the initial atomic state is $|gg\rangle$. However, when the initial atomic state is $|eg\rangle$, a slight increasing of κ makes the period of entanglement disappears and furthermore benefits to generate relative steady and strong entanglement. Although the entanglement may decrease slightly as time evolution, we still can employ the non-period to relieve the restriction of interaction time. In experiment, precisely controlling interaction time is still very difficult. While, it will be not necessary to precisely control the interaction time by employing the slight cavity dissipation. Thus, the dissipation of the cavity is not always bad to the atom-atom thermal entanglement. In some cases such as the initial atomic state $|eg\rangle$, the cavity dissipation is utilizable.

5 Conclusion

To sum up, when two-photon process is involved, we study two atoms simultaneously interact with the thermal field under different couplings and different spontaneous emission rates. To different initial atomic state, the different couplings assist to produce the the atom-atom thermal entanglement in two-photon process. This entanglement is more regular than that of one photon process in sense that in some time intervals the entanglement can survive when difference of two couplings varies in a large range. If the atoms spontaneously emit inevitably, the different spontaneous emission rates is utilizable in generating thermal entanglement. The different spontaneous emission rates can be realized experimentally by localize different atoms in different places in a same F-P cavity. We also investigate the effect of the cavity leakage. To the initial atomic state $|gg\rangle$, the cavity dissipation should be suppressed as possible as we can, but to the initial atomic state $|ee\rangle$, we can employ a slight cavity leakage to relieve the restriction of interaction time so that we can obtain a large and steady entanglement.

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Figure Captions:

Fig. 1a: Two-atom entanglement versus difference of two couplings r and time t for atomic initial state $|ee\rangle$, $N = 1.5$, $\kappa = 0$ and $\Gamma_a = \Gamma_b = 0$.

Fig. 1b: Descriptions are same as in Fig. 1a but for atomic initial state $|gg\rangle$.

Fig. 2a: Two-atom entanglement versus difference of two spontaneous emission rates γ and mean photon number N for atomic initial state $|ee\rangle$, $g = 1$, $r = 0.3$, $\kappa = 0$, $t = 1$ and $\Gamma = 0.2$.

Fig. 2b: Descriptions are same as in Fig. 3a but for $\Gamma = 0.02$, $r = 0.1$ and atomic initial state $|gg\rangle$.

Fig. 3a: Two-atom entanglement versus cavity decay κ and time t for atomic initial state $|gg\rangle$, $g = 1$, $r = 0.1$, $N = 1.5$, $\Gamma_a = \Gamma_b = 0$.

Fig. 3b: Descriptions are same as in Fig. 3a but for atomic initial state $|eg\rangle$.

